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VORTICITY OF TWO-DIMENSIONAL MHD POROUS FLOWS UNDER CROSSED VELOCITY AND MAGNETIC FIELDS

SUNDAR SINGH[1*] AND T. S. CHAUHAN[2]

1. *Department of Physics, Bareilly College, Bareilly (India)-243005*

(Email: ssg01bcb@gmail.com)

2. *Department of Mathematics, Bareilly College, Bareilly (India)-243005*

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Two-dimensional magnetohydrodynamic (MHD) flows of incompressible and homogeneous viscous fluids with infinitely large electrical conductivity through the highly porous network have been considered for analytical studies keeping in mind their wide-ranging practical applications. The steady fluid flow is subjected to a magnetic field perpendicular to the fluid velocity. The technique of hodograph transformation was put into use to transform the nonlinear PDE into a linear one whose solutions represent well known flow types which were analysed for the vorticity. Analytical expressions for various flow parameters including the velocity of fluid, magnetic field, vorticity and streamlines have been obtained. Specifically, about vorticity, it is found that rotational flows possess a constant value for this parameter whereas it is zero for radiating flows.

Keywords : Magnetohydrodynamics, hodograph technique, vorticity, petrochemical engineering, food processing

INTRODUCTION

Porous materials because of their inherent characteristics are highly suitable for their use in ocean engineering, medical appliances, mechanical engineering and several modern industries [1]. Porous media fluid flows are inherently common to the nature and have developed a significant interest among the research community in view of their practical applications in many branches of science and technology. Notable practical applications of such flows include among others their use in food processing industry, chemical processing industry, rotating machines etc. Soil being porous to water seepage and upward water movement for fulfilling the need of plants and trees, makes porous media flows very important in agricultural engineering. In agricultural engineering porous media flows are used for the scientific study of underground water resources and seepage of water in riverbeds.

The knowledge of underground water availability is definitely useful for farmers as they can make a choice for the crops to be cultivated depending upon the water requirement and thus maximize the production and hence their income.

Porous media fluid flows have indispensable role in petroleum technology as such flows are utilitarian in developing the technological understanding of underground movement of natural gas, oil and water though the oil reservoirs. These studies are also beneficial in chemical engineering for the purpose of edifying the processes of purification and filtration of gases and liquids [2]. These are also useful in the drying of bulk materials. Fluid flows through porous networks because of their ubiquitous applications over variegated fields of science and technology have gained significant recognition in academics and research and led to a number of research studies to be carried out by prominent researchers. Notable among them are the works of Gaffney J. J., Pradeep Kumar, Talbot, T. Hayat *et al.* [3-6].

FORMULATION OF THE PROBLEM

A. Khechiba *et al.* have studied the combined MHD and pulsatile flow on porous medium [7]. K. V. S. Raju et al. analyzed MHD convective flow through porous medium in a horizontal channel with insulated and impermeable bottom wall in the presence of viscous dissipation and Joule heating [8]. As per their observations velocity and mean velocity distributions decrease with increase in magnetic parameter M, whereas it decreases with decrease in the porosity parameter. Another notable study about fluid flow through the porous medium is the one by Bommanna Lavanya, who made a study about the MHD rotating flow through a porous medium with heat and mass transfer [9]. Though several studies have been made about the fluid flows in porous media, yet the understanding about such flows is still incomplete which creates the need of their further investigation by researchers and this has also been the motivating factor behind us for the present study.

We have considered 2D streamlined MHD flow of an incompressible and homogeneous viscous fluid through a porous medium with great porosity parameter. The fluid is taken to have infinitely large electrical conductivity. The flow takes place under a uniform magnetic field which is perpendicular to the flow velocity (see fig. 1). Magnetic field has a great influence on the flow of electrically conducting fluids and gives rise to applications such as MHD generators and geothermal energy excitations [10]. A strong external magnetic field at right angled to the flow velocity of an electrically conducting fluid will suppress velocity gradients in the direction of magnetic field [11]. This approximation is called quasi 2D MHD.

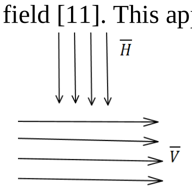

Fig. 1 Schematics of Flow

The flow under transverse magnetic field for infinitely large electrically conducting fluid was investigated by G. Ram and R. S. Mishra [12], who gave the governing equations of the stated problem, which are as follows:

$$\overline{\nabla}.\overline{V} = 0 \qquad \text{... (2.1)}$$

$$\overline{\nabla}.\overline{H} = 0 \qquad \text{... (2.2)}$$

$$\text{Curl }(\overline{V} \times \overline{H}) = 0 \qquad \text{... (2.3)}$$

$$\rho(\overline{V}.\overline{\nabla}) + \frac{\eta}{K}\overline{V} + \eta\nabla^2\overline{V} = \mu(\overline{J} \times \overline{H}) - \overline{\nabla}p \qquad \text{... (2.4)}$$

For two-dimensional flows, we can assume

$$\overline{V} = iV_1 + jV_2 \text{ and } \overline{H} = iH_1 + jH_2 \qquad \text{... (2.5)}$$

Such that

$$q = \left(V_1^{\,2} + V_2^{\,2}\right)^{1/2} \qquad \text{... (2.6)}$$

Since velocity and magnetic fields are perpendicular to each other, therefore from equations (2.3) & (2.4), it can be shown that

$$H_2V_1 - H_1V_2 = A \qquad \text{... (2.7)}$$

$$H_1V_1 + H_2V_2 = 0 \qquad \text{... (2.8)}$$

Here, A is an arbitrary non-zero constant. Solution of equations (2.7) & (2.8) provides magnetic field components as

$$H_1 = -\frac{AV_2}{(V_1^{\,2}+V_2^{\,2})} \;\&\; H_2 = \frac{AV_1}{(V_1^{\,2}+V_2^{\,2})} \qquad \text{... (2.9)}$$

Equation (2.1) gives

$$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} = 0 \qquad \text{... (2.10)}$$

Substitution of equation (2.9) into (2.2) and some rearrangement of terms leads to

$$\left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial V_1}{\partial y} + 2V_1V_2\left(\frac{\partial V_1}{\partial x} - \frac{\partial V_2}{\partial y}\right) + \left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial V_2}{\partial x} = 0 \qquad \text{... (2.11)}$$

We now assume that x & y are functions of velocity components V_1 & V_2 and consider that the Jacobian of V_1 & V_2 with respect to x & y is not zero. We then make use of a very popular technique, called hodograph transformation, to convert nonlinear PDE into linear one. By the use of this technique, equations (2.10) & (2.11) reduce to

$$\frac{\partial y}{\partial V_2} + \frac{\partial x}{\partial V_1} = 0 \qquad \text{... (2.12)}$$

$$\left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial x}{\partial V_2} - 2V_1V_2\left(\frac{\partial y}{\partial V_2} - \frac{\partial x}{\partial V_1}\right) + \left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial y}{\partial V_1} = 0 \qquad \text{... (2.13)}$$

We consider another function $f = f(V_1, V_2)$, such that

$$-\frac{\partial f}{\partial V_1} = y \text{ and } \frac{\partial f}{\partial V_2} = x \qquad \text{... (2.14)}$$

Equation (2.13) then reduces to

$$\left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial^2 f}{\partial V_2^{\,2}} + 4V_1V_2\frac{\partial^2 f}{\partial V_1\partial V_2} - \left(V_2^{\,2} - V_1^{\,2}\right)\frac{\partial^2 f}{\partial V_1^{\,2}} = 0 \qquad \text{... (2.15)}$$

The equation (2.15) acquires a simple form if we use velocity components in their polar forms *i.e.,* $V_1 = q\cos\alpha$ and $V_2 = q\sin\alpha$. In the polar notation, equation (2.15) takes on the form

$$\frac{\partial^2 f}{\partial q^2} - \frac{1}{q^2}\frac{\partial^2 f}{\partial \alpha^2} - \frac{1}{q}\frac{\partial f}{\partial q} = 0 \qquad \text{... (2.16)}$$

Again, if the Jacobian of (x, y) with respect to (V_1, V_2) is not zero, the velocity components can be expressed in terms of x and y. Now using these components, the velocity $\overline{V}$ as given by equation (2.5) has to satisfy equation (2.4), which is actually expressed as

$$\eta\{\overline{\nabla} \times (\overline{\nabla} \times \overline{V})\} + \rho\{(\overline{\nabla} \times \overline{V}) \times \overline{V}\} = -\overline{\nabla} g + \mu(\overline{J} \times \overline{H}) \qquad \text{... (2.17)}$$

Where we have substituted

$$g = p + \frac{1}{2}\rho q^2 \qquad \text{... (2.18)}$$

Remark 1: *For equation (2.17) we have neglected the term* $\frac{\eta}{K}\overline{V}$ *because the permeability K of the porous medium is very high making the stated term negligible compared to the rest of the terms in the equation.*

DIFFERENT TYPES OF FLOW GEOMETRIES

By suitably choosing function f in terms of V_1 & V_2, we can ascertain that some well-known flow is represented by the solution of equation (2.16).

1. Gyrational Flow: We take

$$f = \frac{A_1}{2}\left({V_2}^2 + {V_1}^2\right) + A_2 \qquad \text{... (3.1)}$$

Arbitrary constants $A_1 \neq 0$ and A_2 are such that (3.1) satisfies (2.16) and thus represents one of its solutions.

Equation (3.1) on substitution in (2.14) gives velocity components V_1 and V_2 and using them, we get

$$\overline{V} = iV_1 + jV_2 = \frac{(-iy + jx)}{A_1} \qquad \text{... (3.2)}$$

It can be easily verified that

$$\overline{r}.\overline{V} = 0 \qquad \text{... (3.3)}$$

Thus, the flow represented by (3.1) is a gyrational one with angular velocity $1/A_1$. The streamlines of the flow are represented by

$$\frac{dx}{V_1} = \frac{dy}{V_2} \quad \textit{which gives on integration } x^2 + y^2 = \textit{constant} \qquad \text{... (3.4)}$$

And the vorticity of the flow

$$\omega(x, y) = \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} = \frac{2}{A_1} \qquad \text{... (3.5)}$$

Thus, the flow has a constant vorticity associated with it. Further, it can be easily verified that this kind of flow has a zero current density and a parabolically increasing pressure as

$$p = \frac{\rho r^2}{2{A_1}^2} + A_3 \qquad \text{... (3.6)}$$

2. Radial Flow: Let us now take

$$f = B_1 \tan^{-1} {}^{V_2}\!/_{V_1} \qquad \text{... (3.7)}$$

$$B_1 > 0$$

Again, it can be shown that (3.7) is a solution of (2.16) and the velocity components so determined now give the velocity of the flow as

$$\overline{V} = iV_1 + jV_2 = \frac{(iB_1 \cos\alpha + jB_1 \sin\alpha)}{r} = \frac{B_1}{r}\hat{r} \qquad \text{... (3.8)}$$

As is clear from (3.8) that $\overline{V} \propto \hat{r}$, therefore the flow represented by it is a radial one. Now

$$\frac{dx}{V_1} = \frac{dy}{V_2} \quad i.e. \quad \frac{dx}{{}^{B_1 x}\!/_{r^2}} = \frac{dy}{{}^{B_1 y}\!/_{r^2}} \qquad \text{... (3.9)}$$

Which on integration gives the streamlines of this flow represented by

$$x = B_2 y \qquad \text{... (3.10)}$$

where B_2 is some arbitrary constant. Using the values of velocity components V_1 & V_2, vorticity of the flow can be shown to be zero as follows

$$\omega(x, y) = \frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y} = 0 \qquad \text{... (3.11)}$$

The zero value of vorticity establishes the fact that the radial flow is an irrotational one. The current density associated with this type of flow can be shown to have a constant value $2A/B_1$ and the fluid pressure is a complex function of r.

RESULTS AND DISCUSSION

In this work we have used the technique of hodograph transformations to convert a non-linear flow equation into a linear one (equation 2.16) which represent standard types of flow, the characteristics of which are compared in the table 1:

Table 1: Comparative flow parameters of Gyrational and Radial flow

S. No.	Flow Characteristics	Type of Flow	
		Gyrational	**Radial**
01	Velocity	$\frac{(-iy + jx)}{A_1}$	$\frac{B_1}{r}\hat{r}$
02	Streamlines	$x^2 + y^2 = \text{constant}$	$x = B_2 y$
03	Vorticity	$\omega(x, y) = \frac{2}{A_1} = \text{constant}$	$\omega(x, y) = 0$
04	Current density	$\Omega(x, y) = 0$	$\Omega(x, y) = \frac{2A}{B_1} = \text{constant}$
05	Fluid pressure	$p = \frac{\rho r^2}{2{A_1}^2} + A_3$	p is a complex function of r

By analyzing the table 1 it is concluded that:

For the gyrational flow the velocity is at right angles to the radial direction whereas it is in the radial direction for radial flow. The gyrational flow has streamlines as the concentric circles whereas these are straight lines for the radial type of flow. Further the gyrational flow has a constant (non-zero) vorticity as opposed to a zero value of vorticity for radial flow. However, the radial flow possesses a constant (non-zero) current density but it is zero for the gyrational flow. With regard to the fluid pressure, it is concluded that there exists a parabolically increasing pressure in the case of gyrational flow which, in the case of radial flow, is a complex function of r.

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